B.Sc Math(H), part-1st
paper-I, Group-A
summation of series
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Summation of series ①

Formulae (सूत्र)

① $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots$ to n terms. $\sim \sin\{\alpha+(n-1)\beta\}$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\beta/2}\cdot\sin\left\{\alpha+(n-1)\beta/2\right\} \quad (1) \quad \text{R}$$

sum of Sines of angles in A.P

$$= \frac{\sin\left(\frac{n.\text{difference}}{2}\right).\sin\left(\frac{\text{1st angle}+\text{last angle}}{2}\right)}{\sin\left(\frac{\text{difference}}{2}\right)}$$

Difference का मतलब common difference है।

$$= \frac{\sin\frac{n\beta}{2}}{\sin\beta/2}\sin\frac{1}{2}\left\{2\alpha+(n-1)\beta\right\}$$

[Concept ① को [illegible] $\sin\left\{\frac{2\alpha+(n-1)\beta}{2}\right\}$ [illegible] $= \sin\left(\alpha+\frac{(n-1)\beta}{2}\right)$]

$$= \frac{\sin\frac{n\beta}{2}}{\sin\beta/2}\sin\frac{1}{2}\left\{\alpha+\alpha+(n-1)\beta\right\}$$

② $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots + \cos\{\alpha+(n-1)\beta\}$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\beta/2}\cdot\cos\left\{\alpha+(n-1)\beta/2\right\} \quad \text{R}$$

$x^2 - y^2 = a^2$

$x = a\cosh z$

$y = a\sinh z$

$1+\cos A = 2\cos^2 A/2$

W.O. 4 (2)

$\sin(\pi + A) = -\sin A$, $\sin(2\pi + A) = \sin A$; $\sin(3\pi + \ldots) = -\sin\ldots$ [illegible]

$\sin x - \sin 2x + \sin 3x - \sin 4x + \ldots$

$S = \sin x + \sin(\pi + 2x) + \sin(2\pi + 3x) + \sin(3\pi + 4x) + \ldots$ to n terms

$$= \frac{\sin n\left(\frac{\pi + x}{2}\right)}{\sin \frac{\pi + x}{2}} \cdot \sin\left\{x + (n-1)\frac{\pi + x}{2}\right\}$$

$$= \frac{\sin n\left(\frac{\pi + x}{2}\right)}{\sin\left(\frac{\pi}{2} + \frac{x}{2}\right)} \sin\left\{x + (n-1)\frac{\pi + x}{2}\right\} \text{ Ans}$$

W.O. 5

$\sqrt{1 + \cos\alpha} + \sqrt{1 + \cos 2\alpha} + \sqrt{1 + \cos 3\alpha} + \sqrt{1 + \cos 4\alpha} + \ldots$

$S = \sqrt{2\cos^2\frac{\alpha}{2}} + \sqrt{2\cos^2\alpha} + \sqrt{2\cos^2\frac{3\alpha}{2}} + \sqrt{2\cos^2 2\alpha} + \ldots$

[square root and square cancel out]

$= \sqrt{2}\cos\frac{\alpha}{2} + \sqrt{2}\cos\alpha + \sqrt{2}\cos\frac{3\alpha}{2} + \ldots$ to n terms

$= \sqrt{2}\left[\cos\frac{\alpha}{2} + \cos\alpha + \cos\frac{3\alpha}{2} + \ldots \text{ to } n \text{ terms}\right]$

$$= \sqrt{2}\,\frac{\sin\frac{n\alpha}{4}}{\sin\frac{\alpha}{4}} \cos\left\{\frac{\alpha}{2} + (n-1)\frac{\alpha}{4}\right\} \text{ Ans}$$

$\sin^2 A + \cos^2 A = 1$

$\sin^2(x+y) + \cos^2(x+y) = 1$

$\sin 2A$

$2\sin A \cos A$

$= 2\sin(x+y) \cdot \cos(x+y)$

W.06 (3) $\sqrt{1+\sin 2x} + \sqrt{1+\sin 2(x+y)} + \sqrt{1+\sin 2(x+2y)} + \cdots$

$$S = \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} + \sqrt{\sin^2(x+y) + \cos^2(x+y) + 2\sin(x+y)\cos(x+y)} + \cdots$$

$$S = \sqrt{(\sin x + \cos x)^2} + \sqrt{\{\sin(x+y) + \cos(x+y)\}^2} + \sqrt{\sin(x+2y) + \cos(x+2y)^2} + \cdots$$

Root হটানো sine of angles in A.P. সূত্রানুযায়ী formula প্রয়োগ করে।

$$= (\sin x + \cos x) + \{\sin(x+y) + \cos(x+y)\} + \{\sin(x+2y) + \cos(x+2y)\} + \cdots$$

$$= \{\sin x + \sin(x+y) + \sin(x+2y) + \cdots\} + \{\cos x + \cos(x+y) + \cos(x+2y) + \cdots\}$$

$$= \frac{\sin \frac{ny}{2}}{\sin \frac{y}{2}} \sin\left\{x + (n-1)\frac{y}{2}\right\} + \frac{\sin \frac{ny}{2}}{\sin \frac{y}{2}} \cos\left\{x + (n-1)\frac{y}{2}\right\}$$

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7 (04) Find the sum of

W.O $\sin\alpha\sin(\alpha+\beta)-\sin(\alpha+\beta)\sin(\alpha+2\beta)+\sin(\alpha+2\beta)\sin(\alpha+3\beta)-\ldots$ to $2n$ terms

[concept: $\sin\alpha\cdot\sin(\alpha+\beta)$ ... terms product हैं [illegible] L.H.S [illegible] R.H.S [illegible]]

Let $S=\sin\alpha\sin(\alpha+\beta)-\sin(\alpha+\beta)\sin(\alpha+2\beta)+\sin(\alpha+2\beta)\cdot\sin(\alpha+3\beta)-\ldots$ to $2n$ terms

$\therefore 2S=2\sin\alpha\cdot\sin(\alpha+\beta)-2\sin(\alpha+\beta)\sin(\alpha+2\beta)+2\sin(\alpha+2\beta)\sin(\alpha+3\beta)-\ldots$ to $2n$ terms

or $2S=\cos\beta-\cos(2\alpha+\beta)-\{\cos\beta-\cos(2\alpha+3\beta)\}+\{\cos\beta-\cos(2\alpha+5\beta)-\ldots$ to $2n$ terms $\}$

$[2\sin A\sin B=\cos(A-B)-\cos(A+B)]$

$2S=\{\cos\beta-\cos\beta+\cos\beta-\cos\beta+\ldots 2n \text{ terms}\}-[\cos(2\alpha+\beta)-\cos(2\alpha+3\beta)+\cos(2\alpha+5\beta)-\ldots \text{to } 2n \text{ terms}]$

or, $2S=0-[\cos(2\alpha+\beta)+\cos(\pi+2\alpha+3\beta)+\cos(2\pi+2\alpha+5\beta)+\ldots \text{to } 2n \text{ terms}]$

or $2S=-\dfrac{\sin 2n\frac{\pi+2\beta}{2}}{\sin\frac{\pi+2\beta}{2}}\cos\left\{(2\alpha+\beta)+(2n-1)\frac{\pi+2\beta}{2}\right\}$

or $S=-\dfrac{\frac{1}{2}\sin n(\pi+2\beta)}{\sin(\pi/2+\beta)}\cos\left\{2\alpha+\beta+\left(n-\tfrac{1}{2}\right)(\pi+2\beta)\right\}$

$=-\dfrac{1}{2}\dfrac{\sin n(\pi+2\beta)}{\cos\beta}\cos\left(2\alpha+\cancel{\beta}+n\pi+2n\beta-\frac{\pi}{2}-\cancel{\frac{2\beta}{2}}\right)$

$=-\dfrac{1}{2}\dfrac{\sin n(\pi+2\beta)}{\cos\beta}\cos\left\{2\alpha+2n\beta+\left(n-\tfrac{1}{2}\right)\pi\right\}$ Ans